An Optimal Approach to Project Selection

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Paper # 165

Introduction: The Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP), developed by Thomas Saaty [1] in the 1970s, provides a general framework for organizing and prioritising the various elements of any type of decision. It has been used prolifically in contexts such as recruitment, oil exploration, strategic planning, risk management and project selection, among many others. It is best applied to complex decisions with multiple-criteria and sub-criteria, whence the hierarchical character of the method. It provides many benefits to the decision-making team, including a focal point for deliberations, a framework for applying and testing corporate values and preferences and an umbrella under which other decision-support systems can feed their results seamlessly. It also provides a means for collaborative and distributive decision-making, yielding efficiency and quality gains simultaneously.

The process begins with the establishment of the criteria tree, and the identification of a set of viable decision options. Next relative preference rating assessments are made for each option, (known collectively as a preference vector), in relation to each of the lowest level sub-criteria in the hierarchy. For example, if there are four options, and each is rated equally against a criterion, this would appear as [25%, 25%, 25%, 25%]. A preference vector relating to a different criterion might appear as [40%, 10%, 30%, 20%], showing the first to be the most preferable. Note that the vector is normalised here to sum to 100%. These assessments can be arrived at by a variety of means, including subjective evaluation, pair-wise comparison techniques, voting process or transformation from external analyses.

The method also requires the relative evaluations of the criteria against each other at each level. Preference vectors for the decision options against higher level criteria are then calculated as a weighted average of the preference vectors corresponding to the level below, the weights being provided by the relative sub-criteria preferences at that lower level. This process is continued up the hierarchy until a vector if found for the decision itself at the top of the tree. In ordinary usage, the top rating system in this vector is regarded as providing the best overall choice, subject to the criteria considered and the weightings supplied.

A sample hierarchy is shown in figure 1 below. Here a decision to invest in either property or equity is considered against three high-level criteria: Return, Social Responsibility and Security. The first two are decomposed into sub-criteria as shown. Table 1 shows the assessed or calculated vectors representing the relative appeal of the decision options against the lowest level sub-criteria and also the relative importance of these criteria against each other. The calculated vectors are shown in table 2.

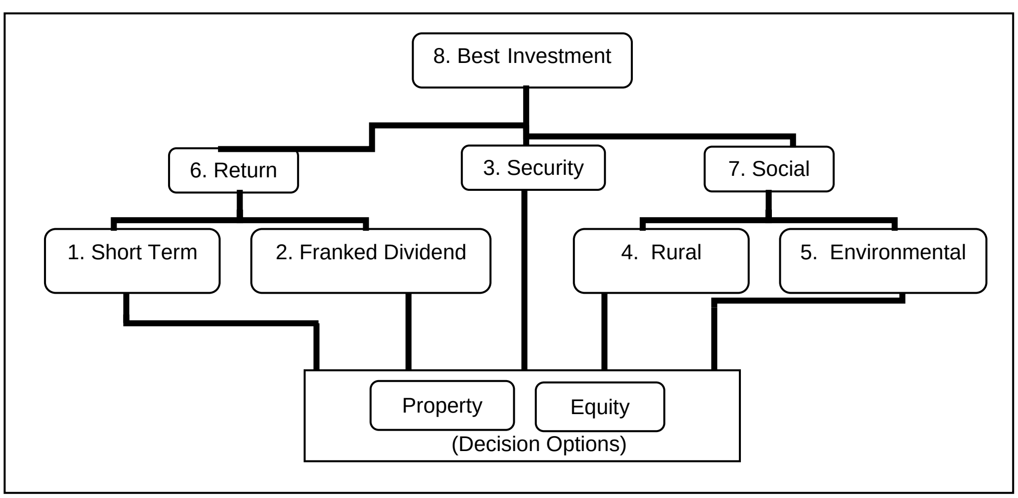

Figure 1: Decision Structure

No.	Issue	In relation to…..	Normalised
	Decision Options		
1	Property vs Equity	Short tem	[15%, 85%]
2	Property vs Equity	Franked Dividends	[0%, 100%]
3	Property vs Equity	Security	[75%, 25%]
4	Property vs Equity	Rural Focus	[25%, 75%]
5	Property vs Equity	Environmental Focus	[10%, 90%]
	Lowest Level Criteria		
6	Short Term vs Franked Divs.	Return on Investment	[67%, 33%]
7	Rural vs Environmental	Social Responsibility	[50%, 50%]
	Higher Level Criteria		
8	Return vs Social vs Security	Decision Objective	[40%, 40%, 20%]

Table 1: Input Data

No.	Decision Result	In relation to…..	Normalised
1	Property vs Equity	Return	[10%, 90%]
6	Property vs Equity	Social	[17.5%, 82.5%
7	Property vs Equity	Best Investment	[37.5%, 62.5%]

Table 2: Calculated Results

The vectors in the final column of table 2 show the calculated relative desirability of the two options at the remaining nodes of the tree, the last representing the top node and therefore the result of the entire decision itself. We see here that 'Property' scores higher than 'Equity' as is therefore the preferred option.

Constrained Strategic Value Maximisation

When the AHP is applied to the problem of project selection at a strategic level, it can yield a subset of candidate projects rather than a single choice only, as was the case in the example above. The selection of the candidates is governed by maximising the sum of their preference ratings (their strategic value), while constraining their aggregate consumption of some common resource, usually cost, to remain within some prescribed constraint. The treatment of cost as a global resource common to all projects (rather than as one of the criteria in each decision), allows for an integrated selection process to occur.

In what follows, we will describe how the project selection process can be formulated. The approach will echo that found in [2] and similar texts, which recognises the need to make tentative decisions based upon uncertain information, and to use the corresponding results as a guide for further refinement.

We shall begin by providing broad cost estimates of each project, and identify the subset of most attractive projects (in terms of their combined strategic value) that meet the cost constraints. These will be considered as best candidates for further cost refinement, which when performed will provide new cost data, and in turn used to identify possibly a different selection subset. In this way, each successive solution informs the next with increasing certainty, all the while limiting costing details to levels commensurate with the degree of confidence currently held for their ultimate selection. This continues until the selection subset converges to one immune to further cost changes, or all uncertainties have been removed. Extensions of this model to accommodate the effects of risk will be provided.

An Integer Programming (IP) Formulation

Consider the case where N strategic directions are being considered, each represented by a project or program with a corresponding current approximate cost of C_j, j = 1…N.

The problem is then to find the set of selection variables X_j , each taking on a value of one or zero (where one implies the selection of project j and zero implies its rejection), which maximises the strategic value

$\mathbf{V} = \Sigma\, \mathbf{X}_j V_j$

where V_j is the preference value for option j, j = 1…N in the top-level preference vector in the AHP formulation,

subject to the global resource constraint

$\Sigma\, X_j C_j <= \mathbf{C}$

where C is a prescribed aggregate cost constraint.

V will therefore be some value less than or equal to 100%, representing the percentage of total value that can be attained within the budget.

It is possible to add further constraints to the model to reflect interdependencies between the options. For example:

1. $X_b = 1$ Option b must be selected
2. $X_b = 0$ Option b must not be selected
3. $X_b + X_c = 1$ One and only one of either option b or c must be selected.
4. $X_b + X_c <= 1$ Options b and c should not be selected together.
5. $X_b <= X_c$ Option b implies the selection of option c
6. Xb = Xc Either both or neither of options b and c must be selected

This problem can be modelled by means of a zero-one integer programming formulation and is easily implemented within in Microsoft Excel™ by means of the 'Solver' add-in.

Monte Carlo and Iterative Selection

While the IP formulation delivers a selected subset, the sensitivity of its constitution to uncertainties in the cost estimate is unknown. This suggests:

1, The need for a Monte Carlo simulation which samples a large number of random costs distributed around the current estimate and solves the IP problem for each. When the results are aggregated, this provides an indication of the probability of project j being selected.

2. An iterative approach which uses these selection probabilities as an indicator of the projects for which refinement of cost estimates would be best served for a new solution at the next step.

Specifically, for the Monte Carlo process we select a random value from the likely distribution of variations in cost estimates around C_j, j = 1 …N. Let C_{ji} be the random value in Monte Carlo run i (i = 1 to M). At each run, we solve the IP problem with these $C_{ji,}$ producing selection variables X_{ji} again taking on values of zero or one.

For any run i, the total cost across all projects is $\Sigma X_{ji}C_{ji}$, j = 1 to N and the total strategic value produced is $\Sigma X_{ji}V_j$

What we require is a summary or representative value for the selection variables across the M samples at Monte Carlo run number i. This can be obtained as follows:

For any project j, the average cost across all runs will be

$C'_j = [\Sigma X_{ji}C_{ji}]/M$, i = 1 to M

If project j were to be selected in every run, all X_{ji} s for it would be equal to one and the average cost across all samples of the run would be

$C'_j = \Sigma C_{ji}/M$, i = 1 to M

But, since C_{ji} s are unbiased estimates of C_j we would expect that

$C_j = \Sigma C_{ji}/M$

so that if project j were selected in every run, we would expect

$C'_j = C_j$

It follows that the ratio

$Y_j = C'_j/C_j$

provides a measure of the frequency with which project j is selected (a "selectability" index for project j. Therefore Y_j is a non integral ('fuzzy') approximation of X_j, which will approach either zero or one as the uncertainty in the cost estimate decreases. It can be interpreted as the current probability of project j being ultimately selected

Projects with Y_j values relatively close to one (say greater than 0.5) therefore appear to have greater appeal in terms of their cost/benefit value than the others and therefore, at run i, make claim for higher priority in terms of cost estimate refinement. This might result in a new value for C_j and with a smaller associated uncertainty interval.

This introduces then need for an iterative approach since these values can be fed into a new Monte Carlo simulation (again with M runs) which in turn will produce a new set of Y_j s. Depending upon the sensitivity of the system, this might yield a newly constituted subset of likely selection candidates. The analyst can also intervene manually by forcing certain selections or exclusions (setting the corresponding selection variable to one or zero as required). The process is then repeated until there is sufficient confidence that the selection subset has stabilised. It is shown in the diagram below.

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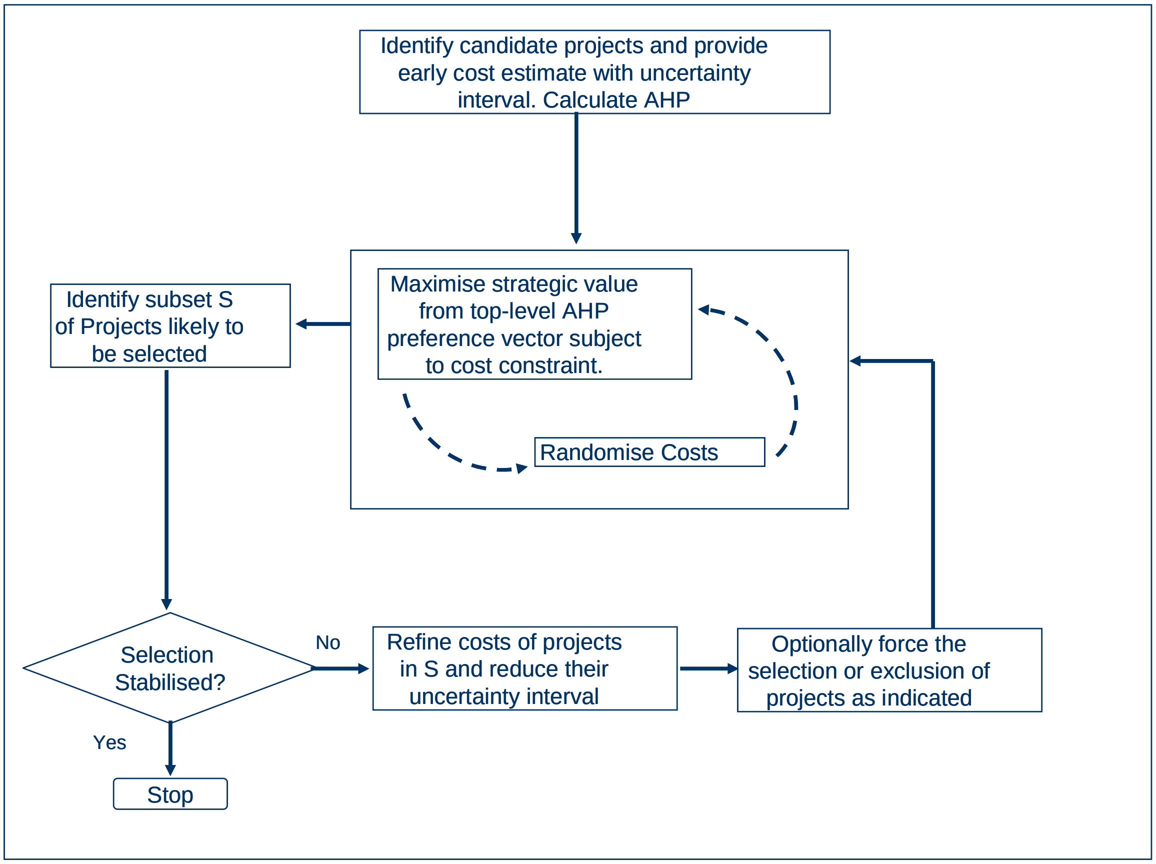
Identify candidate projects and provide early cost estimate with uncertainty interval. Calculate AHP
Maximise strategic value from top-level AHP preference vector subject to cost constraint.
Randomise Costs
Identify subset S of Projects likely to be selected
Selection Stabilised?
No
Yes
Stop
Refine costs of projects in S and reduce their uncertainty interval
Optionally force the selection or exclusion of projects as indicated

Figure 2

Sample Calculations

A problem with ten projects was run. The initial costs, AHP decision result and uncertainty intervals for the first step are shown in the first row of table 3. As costs for the currently selected projects are refined so the overall uncertainty in the system reduces. The sequence of selection subsets is shown in table 4 below. Underlined values indicate a change from the previous step.

Run	**Project Initiative No.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	Strategic Value	13.									
1	Uncertainty radius	30	40	25	30	15	25	20	40	30	20
	Cost Estimate	2.9	7.5	1.7	6.5	2.8	6	2	5.9	5.8	8.4
2	Uncertainty radius	**10**	40	10	10	15	25	20	40	30	20
	Cost Estimate	2.9	7.5	1.7	**9.5**	3.5	6	2	5.9	5.8	8.4
3	Uncertainty radius	**5**	40	**5**	**5**	**5**	25	**5**	**20**	30	20
	Cost Estimate	**3.2**	7.5	**2**	**10**	3	6	2	**5.4**	5.8	8.4
4	Uncertainty radius	5	40	5	5	5	25	5	**5**	30	20
	Cost Estimate	3.2	7.5	2	**11**	3	6	2	**5**	5.8	8.4
5	Uncertainty radius	0	**5**	5	5	5	25	5	5	30	20
	Cost Estimate	3.2	**8.5**	2	11	3	6	2	5	5.8	8.4

Table 3 – Sample Results

Run No.	**Selected Subset**	**Fuzzy Cost**	**Strategic Value**
1	{1,3,4,5,7,8}	23.9	78.27
2	{1,3,4,5,7,8}	23.7	66.503
3	{1,3,4,5,7,8}	23.7	66.11
4	{1,2,3,5,7,8}*	22.9	65.35
5	{1,2,3,5,5,7,8}	23.7	65.74
6	{1,2,3,5,5,7,8}	23.7	65.76

Table 4-
Sequence of selection subsets and corresponding costs and values.

Row three in this table shows a change with project 4 leaving the selection subset due to a significant increase in costs. It was replaced by project 2. No further changes were found in the selection sequence.

Incorporating Risk

The initial AHP decision formulation is designed to reflect positive criteria such as strategic alignment to the business plan, profitability, corporate citizenship and social responsibility, customer satisfaction and the like. However, it does not yet account for risk. Like cost, risk can be considered as a global resource, to be shared across projects and therefore in limited supply. It is not therefore appropriate to treat risk as merely another criterion in the hierarchy.

One approach [1] is characterised by performing a separate but similar AHP solution process, one that focuses purely on negative criteria reflecting identified risk factors directly. These might include occupational health and safety, alienation of share-holders, technical failure, service provider dependence and many more. Care should be taken to insure that risk criteria are not simply the inverse of the positive ones already considered. The problem with this approach is that there is no rational way by which the two solutions can be compared.

We can integrate risk considerations into the scheme described above in two ways:

1. Trade risk for cost by identifying the approximate cost of reducing the risk for each project and adding this to the project costs C_j above. Again the process would be driven by successively more detailed risk assessments as the likelihood of selection increases.

2. Quantify the aggregate risk (or residual risk if step 1 is taken also) on some common scale for each project. This can be done by a number of means, the simplest perhaps by means of a linear function of the quantitative assessments of likelihood and consequences for each risk factor and summing these for each project to form the value R_j, j = 1 … N. We can then compute the risk inherent in the selection at any iteration as:

$\Sigma Y_j R_j$

If this is done within the process outlined earlier, a map of cost/benefit values vs Risk can be produced to aid in the production of the sequence of selection subsets. This could appear as follows.

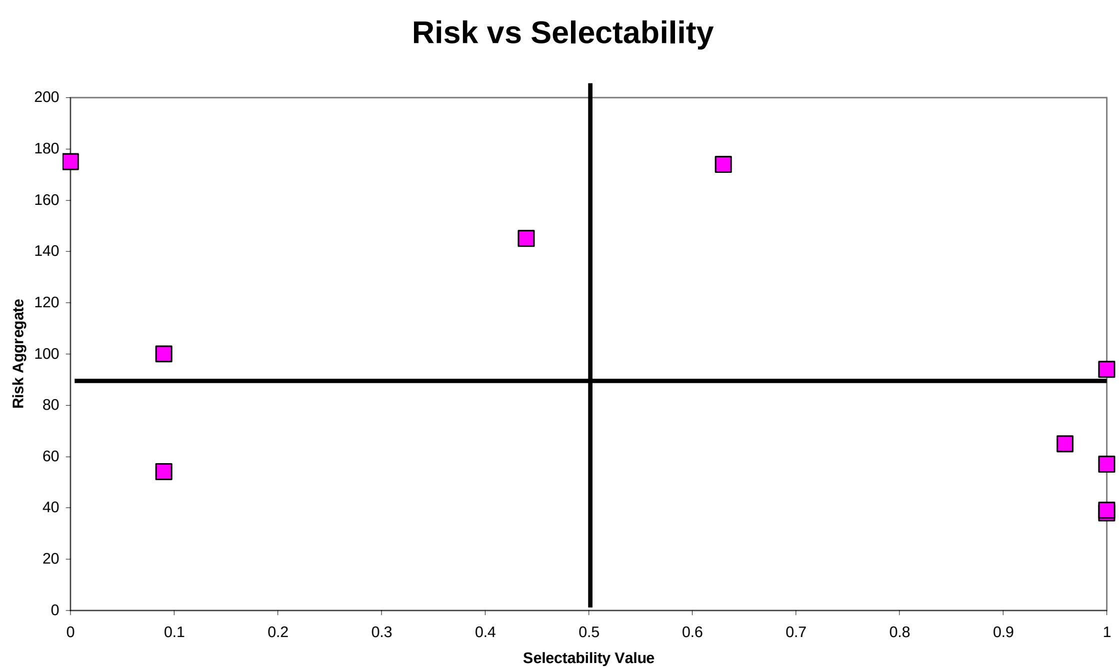

Figure 3

Here the projects of greatest interest would be those in the lower right hand quadrant, corresponding to high selectability and lower risk. If the analyst determines that any of these risks are unacceptably high, the corresponding project can be manually removed from the selection subset. However, it is preferable to allow the optimisation process to make this decision by adjust it to accommodate maximum tolerable risk. This can be achieved by simply adding to our IP formulation a constraint of the form

$$\Sigma X_j R_j <= \mathbf{R_{max}}$$

where $\mathbf{R_{max}}$ is the maximum risk deemed acceptable.

The series of selections will each now reflect not only cost/benefit appeal but also the amount of risk exposure entailed. The process continues as before, but now with refinements to both risk and cost made to currently selected project and the corresponding narrowing of their uncertainty intervals until a stable result is obtained.

The revised process is shown in the diagram in figure 4 below.

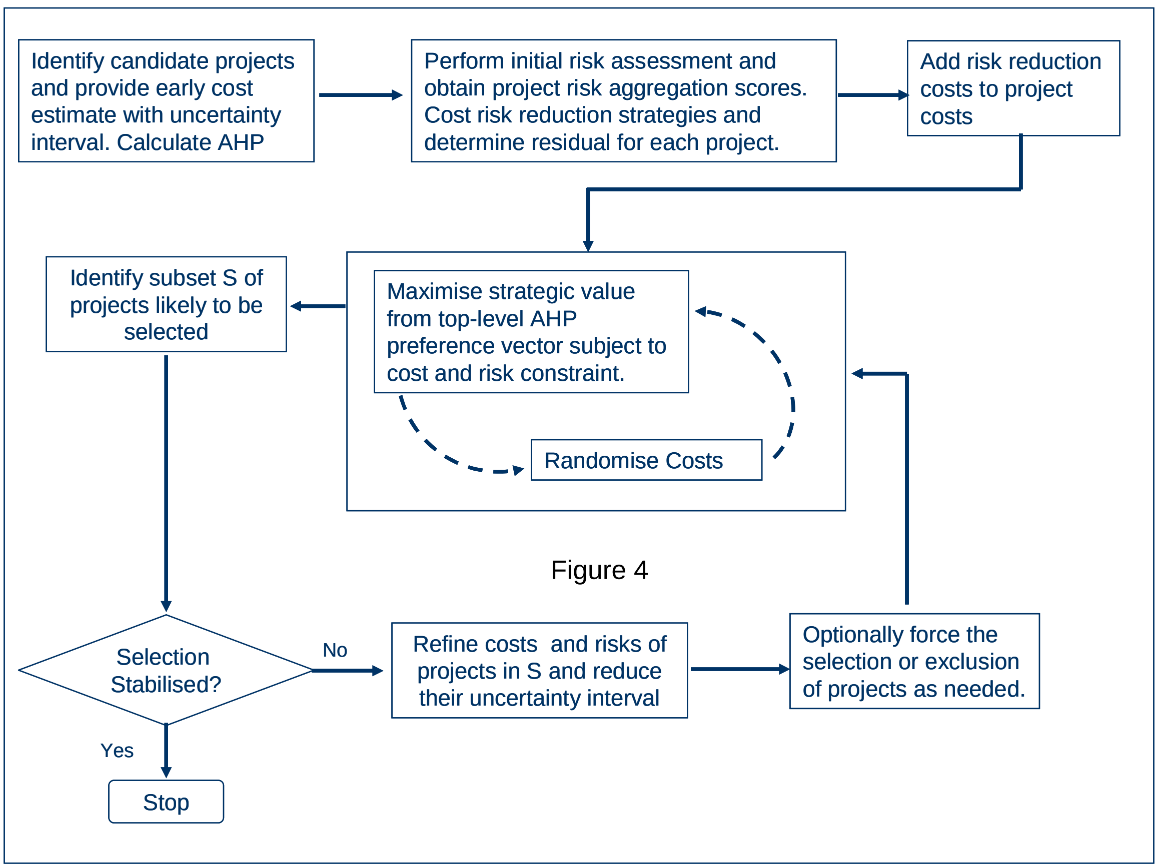

Figure 4

Practical Issues

This process is intrinsically collaborative and distributive, with program managers orchestrating the costing effort for individual project initiatives, each operating under the direction of a strategic officer who will inform them of refinement and uncertainty requirements. Responsibilities will mirror the hierarchy, with more experienced personnel occupying higher positions, and junior staff obtaining valuable experience at the lower echelons of the tree.

Although the dynamic nature of this process appears confined to the strategic value optimisation, there is no reason that the AHP calculations remain static. Various AHP criteria and option evaluations can be tightened and the iterative process restarted in order to assure a more robust analysis.

It is of course of course possible for the process to ignore projects that are ultimately deserving of selection. This could happen under one or more of the following conditions:

- Poor evaluations were made for those projects within the AHP formulation.
- Poor cost estimates were made for them
- Insufficient uncertainty was allowed to cover for the poor estimate.

These should be kept in mind in relation to projects that are persistently ignored by the process.

Conclusions

The combination of processes for decision-making, project costing, risk quantification and strategic value optimisation provide both power and flexibility for the problem of project selection. Further flexibility is made available by the staged approach inherent in the overall process, allowing for management intervention in the form of forced selection or exclusion of certain projects for reasons that might defy formulation within the model.

Further research here is needed on questions of appropriate risk aggregations, and perhaps a refinement on the selection process at each iteration of the procedure.

References

[1] Thomas L Saaty, 1982. Decision Making for Leaders: The Analytical Hierarchy Process for Decisions in a Complex World, Lifetime Learning Pub.

[2] Doug DeCarlo , 2004. eXtreme Project Management: Using Leadership, Principles, and Tools to Deliver Value in the Face of Volatility, Jossey-Bass.

[3] Thomas P. McAuliffe, 2005. The 90% solution – A Consistent Approach to Optimal Business Decisions, Author-House.